Evaluation of Aerodynamic Derivatives from a Magnetic Balance System

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Theme

IT is demonstrated that it is feasible to extract static and dynamic aerodynamic derivatives from six-degree-of-freedom motion data for a 15° cone at Mach 3. Two methods of solving the inverse problem, both of which allow the inclusion of a large class of aerodynamic nonlinearities, are compared and the influence of noise on extraction accuracy is studied.

Contents

When it comes into operation, the University of Virginia (three component) magnetic balance wind-tunnel facility will support a magnetic sphere in a Mach 3 wind tunnel.[1] A model, built around the magnetic sphere, is essentially rotationally free at all frequencies and is free in translation at frequencies above some adjustable "balance frequency" (of the order of 10–15 Hz). The system, by responding to translation at frequencies below the balance frequency, holds the model in the test section. It is straightforward to arrange a passive, static magnetic restoring moment on the model so that it oscillates in rotation and translation at a frequency well above the balance frequency. Thus stable motion of an aerodynamically unstable model can be arranged.

The study of the dynamic stability (or the evaluation of the dynamic derivatives) of a model corresponds to measuring the balance forces and moments, observing the model motion, and inverting the equations of motion to obtain both static and dynamic derivatives of the model. The motion of the model cannot be restricted to simple one or two-degree-of-freedom oscillations. Consequently, it was considered important to study the character of the problem of extracting aerodynamic derivatives from six-degree-of-freedom motion data.

The analysis proceeded by calculating the motion of a 15° cone, the anticipated first model, in a reasonably realistic approximation of the model in the tunnel and using the full six-degree-of-freedom equations of motion. The z force (in body frame) and pitching moment derivatives (and, of course, the corresponding lateral derivatives which follow from symmetry of the body)[2,3] which have nonzero values are the following:

$$C_{K_w} = \partial C_K/\partial(W/V_T), \quad C_{K_q} = \partial C_K/\partial(Q\,d/V_T) \qquad K = z, m$$

$$C_{K_{D\omega}} = \partial C_K/\partial\left(d\frac{\partial W}{\partial t}\Big/ V_T^2\right), \quad C_{K_{pv}} = \partial C_K/\partial(P\,dV/V_T^2) \qquad K = z, m$$

C_z = force coefficient, C_m = pitching moment coefficient, d = base diameter, P, Q, R = angular velocities about body fixed axes, U, V, W = translational velocities in body fixed axes, V_T = tunnel speed.

The motion thus generated, called perfect data, consists of sets of values (100 sets) of (x, y, z) the position of the center of mass in the tunnel-fixed axis system and (ψ, θ, ϕ) the conventional orientation angles (about 4 cycles in ψ and θ). Noisy data are simulated by adding sets of pseudo-random numbers with zero mean and various standard deviations $(\sigma_x, \sigma_y, \sigma_z, \sigma_\psi, \sigma_\theta, \sigma_\phi)$ to the perfect data.

The inverse problem to extract aerodynamic derivatives from the observed motion was solved by two methods: the "Brute-force" method and the method of parametric differentiation. Both methods have the following features: 1) the inherent nonlinear character of the inertia terms in the equations is retained, and 2) they do not depend on an analytic solution of the equations of motion. Further, Brute-force can handle any sort of aerodynamic nonlinearity and parametric differentiation can handle a large class of aerodynamic nonlinearities. This capability is considered to be very important since the UVa system probably has the potential of investigating experimentally nonlinear aerodynamics.

1) The Brute-force method treats the equations of motion as linear algebraic equations in the unknown aerodynamics derivatives and uses the method of least squares to average over a large number of data points. The required velocities and accelerations in the equations of motion are calculated as follows: a) the noisy data are smoothed by using 9 equally spaced points and a quintic power series to calculate one central smoothed data point, b) the smoothed data are numerically differentiated using 5 equally spaced points, and c) the appropriate quantities are transformed to the body fixed axis system.

2) The method of parametric differentiation considers the equations of motion imbedded in a parametric space of initial conditions and aerodynamic derivatives. The aerodynamic derivatives are evaluated by a least squares fitting of the observed motion to a calculated motion by adjusting the parametric coefficients. Chapman and Kirk have considered the angular motion of missiles by this method.[4]

To verify the inversion methods and to make a first attempt to understand the modelling problem, the perfect data, uncorrupted with noise, were treated first. For both methods the number and order of aerodynamic derivatives chosen to be extracted are arbitrary. However, in the parametric differentiation computer code, largely due to its complexity, the order and maximum number of the derivatives to be extracted were fixed and therefore not optimized. The tentative conclusions from this initial study were that 1) both methods of inversion would work and 2) for the 15° cone two groupings of each of the force and moment damping derivatives were important to a description of the motion and could be extracted with reasonable accuracy. The full report describes these preliminary results in some detail.

The influence of noise in the data on extraction accuracy was considered, for practical design reasons, to be a primary objective of the analysis. The sets of derivatives which seemed to be extracted best for each method treating the perfect data were chosen for the noise study (C_{m_w}, C_{m_q}, $C_{m_{Dw}}$, C_{z_w}, C_{z_q}, and $C_{z_{Dw}}$ for Brute-force; C_{m_w}, C_{m_q}, $C_{m_{pv}}$, $C_{m_{Dw}}$, C_{z_w}, C_{z_q}, and $C_{z_{pv}}$ for

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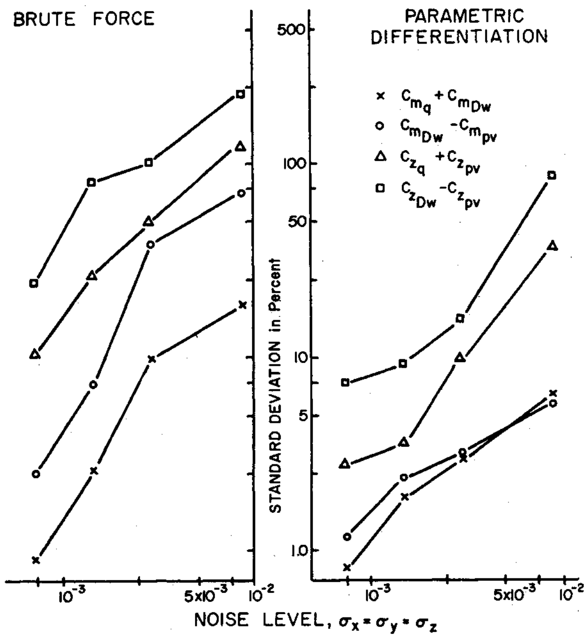

Fig. 1 Standard deviation in percent of the extracted derivative groupings as a function of noise level. The highest noise corresponds to σ being about 10% of the amplitude of the typical variable.

parametric differentiation). At each noise level 5 independent sets of noise were used as a sample. For each case the noise levels in the translational positions x, y, z were the same and three times as large as the noise in the angular positions ψ and ϕ. No noise was added to the roll position ϕ. The standard deviations in percent of the extracted static derivatives and groupings of the damping derivatives were taken as measures of the influence of noise. The maximum noise level $\sigma_x = \sigma_y = \sigma_z$ was about 10% of the amplitude of a typical variable.

Table 1 and Fig. 1 present a summary of the noise results. It is not surprising that the static force and moment derivatives are obtained with significantly greater accuracy than the damping derivatives. It is somewhat surprising that the method of parametric differentiation extracts C_{z_w} with greater accuracy than it extracts C_{m_w}. An over-all view of Fig. 1 shows that the method of parametric differentiation performs better in extracting the significant groupings of force and moment damping derivatives than does Brute-force. Again it is perhaps surprising that both methods, especially parametric differentiation, extract the force damping grouping as well as they do. Though the computer cost for parametric differentiation is about 25 times that of Brute-force (the latter is a quite simple code) the cost of acquiring good experimental data is such that one envisions using virtually all available methods of information extraction.

Table 1 Standard deviation in percent vs noise level for static derivatives

Noise level	Brute-force		Parametric differentiation	
$\sigma_x = \sigma_y = \sigma_z$	C_{Z_w}	C_{m_w}	C_{Z_w}	C_{m_w}
0.00075	0.254	0.167	0.096	0.350
0.0015	1.036	0.560	0.096	0.742
0.003	1.354	1.118	0.229	1.135
0.009	8.763	2.308	1.057	2.214

The chosen "best" values of derivatives used to calculate the perfect data

C_{m_w}	C_{m_q}	$C_{m_{Dw}}$	$C_{m_{pv}}$	C_{z_w}	C_{z_q}	$C_{z_{Dw}}$	$C_{z_{pv}}$
−0.468	−2.94	−1.334	−0.04	−1.85	−2.81	−2.34	−0.052

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